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Spectral functions for the flat plasma sheet model

I G Pirozhenko[1]

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia

E-mail: pirozhen@theor.jinr.ru and irina@spectro.jussieu.fr

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Abstract

The present work is based on Bordag M *et al* 2005 (*J. Phys. A: Math. Gen.* **38** 11027) where the spectral analysis of the electromagnetic field on the background of an infinitely thin flat plasma layer is carried out. The solutions to Maxwell equations with the appropriate matching conditions at the plasma layer are derived and the spectrum of electromagnetic oscillations is determined. The spectral zeta function and the integrated heat kernel are constructed for different branches of the spectrum in an explicit form. The asymptotic expansion of the integrated heat kernel at small values of the evolution parameter is derived. The local heat kernels are considered also.

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1. Introduction

Recently Barton has studied [2] infinitesimally thin two-dimensional plasma layers with flat and spherical geometry roughly reproducing the single base plane from graphite and the giant carbon molecule C_{60}.

The model of the plasma layer is described by the Maxwell equations with charges and currents distributed along the surface Σ:

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{E} - \mathrm{i}\omega \mathbf{B}/c = 0, \tag{1}$$

$$\nabla \cdot \mathbf{E} = 4\pi\delta(\mathbf{x} - \mathbf{x}_\Sigma)\sigma, \qquad \nabla \times \mathbf{B} + \mathrm{i}\omega \mathbf{E}/\mathrm{c} = 4\pi\delta(\mathbf{x} - \mathbf{x}_\Sigma)\mathbf{J}/\mathrm{c}. \tag{2}$$

The time variation of all the dynamical variables is defined by a common factor $\mathrm{e}^{-\mathrm{i}\omega t}$. The surface charges and electric currents are determined by the tangential components of electric field $\sigma = e^2 n_0/(m\omega^2)\nabla_\parallel \cdot \mathbf{E}_\parallel$, $\mathbf{J} = \mathrm{i}e^2 n_0/(m\omega)\mathbf{E}_\parallel$, where n_0 is the equilibrium electron density.

[1] Present address: Laboratoire Kastler Brossel, UPMC, case 74, Campus Jussieu, F-75252 Paris Cedex 05, France.

To put it differently, outside the plasma layer the Maxwell equations without sources are satisfied, whereas approaching the singular surface Σ the fields meet the following matching conditions:

$$[\mathbf{E}_\parallel] = 0, \qquad [\mathbf{E}_\perp] = 2q(c/\omega)^2 \nabla_\parallel \cdot \mathbf{E}_\parallel, \tag{3}$$

$$[\mathbf{B}_\perp] = 0, \qquad [\mathbf{B}_\parallel] = -\mathrm{i}q(c/\omega)\mathbf{n} \times \mathbf{E}_\parallel. \tag{4}$$

Here q is a characteristic wave number $q = 2\pi ne^2/mc^2$, the square brackets $[\mathbf{F}]$ denote the discontinuity of the field $\mathbf{F}$ when crossing the surface Σ, and $\mathbf{n}$ is a unit normal to this surface.

The present work is devoted to the spectral analysis of the plasma sheet model with the simplest flat geometry of the plasma layer.

2. Solution to the Maxwell equations for a flat plasma sheet

Let us take the plasma layer as the coordinate plane $\mathbf{s} = (x, y)$, the axes z being normal to this plane; $\mathbf{k}$ is a two-component wave vector parallel to the plasma sheet, and $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ are unit base vectors in this coordinate system.

It is convenient to express the fields in terms of electric $\mathbf{\Pi}' = \mathbf{e}_z\, \mathrm{e}^{\mathrm{i}\mathbf{ks}}\Phi(z)$ and magnetic $\mathbf{\Pi}'' = \mathbf{e}_z\, \mathrm{e}^{\mathrm{i}\mathbf{ks}}\Psi(z)$ Hertz vectors each possessing one nonzero component [3]

$$\mathbf{E} = \nabla \times \nabla \times \mathbf{\Pi}', \qquad \mathbf{B} = -\mathrm{i}\frac{\omega}{c}\nabla \times \mathbf{\Pi}' \quad \text{(TM-modes, } B_z = 0\text{)}; \tag{5}$$

$$\mathbf{E} = \mathrm{i}\frac{\omega}{c}\nabla \times \mathbf{\Pi}'', \qquad \mathbf{B} = \nabla \times \nabla \times \mathbf{\Pi}'' \quad \text{(TE-modes, } E_z = 0\text{)}. \tag{6}$$

Then the Maxwell equations are reduced to one-dimensional equations for the scalar functions $\Phi(z)$ and $\Psi(z)$

$$-\Phi''(z) = \left(\frac{\omega^2}{c^2} - k^2\right)\Phi(z), \qquad -\Psi''(z) = \left(\frac{\omega^2}{c^2} - k^2\right)\Psi(z),$$
$$-\infty < z < \infty, \qquad z \neq 0, \qquad 0 \leqslant k^2 < \infty. \tag{7}$$

The matching conditions (3), (4) for the fields on Σ lead to the matching conditions for the functions $\Phi(z)$ and $\Psi(z)$ at the point $z = 0$:

$$[\Phi(0)] = -2q\left(\frac{c}{\omega}\right)^2\Phi'(0), \qquad [\Phi'(0)] = 0, \tag{8}$$

$$[\Psi(0)] = 0, \qquad [\Psi'(0)] = 2q\Psi(0), \tag{9}$$

where the notation is introduced $[F(z)] = F(z+0) - F(z-0)$.

The spectral problem for the TE-modes (7), (9) is a self-adjoint one and can be reformulated in another way by introducing the δ-function potential [4]

$$-\Psi''(z) + 2q\delta(z)\Psi(z) = p^2\Psi(z), \qquad \frac{\omega^2}{c^2} = k^2 + p^2. \tag{10}$$

The integration of this equation over z from $-\epsilon$ to ϵ gives in the limit $\epsilon \to 0$ the matching conditions (9).

The spectral problem for the TM-modes bears a resemblance to the problem with the δ' potential, however it is not the case [5]. The matching conditions for Φ involve the eigenvalues of the initial three-dimensional spectral problem (1). This implies that the dynamics of TM-modes along the z axes 'feels' the parallel dimensions (x, y). Obviously, the self-adjointness condition is not satisfied here.

It is important to note that we shall consider such functions $\Phi(z)$ and $\Psi(z)$ which either oscillate ($p^2 \equiv \omega^2/c^2 - k^2 > 0$) or decrease ($\kappa^2 \equiv k^2 - \omega^2/c^2 > 0$) when $|z| \to \infty$. In the first case we are dealing with the scattering states and in the second one the solutions of the Maxwell equations are related to the surface plasmon.

The scattering states for the TE-modes are described by the functions

$$\Psi(z) = C_1\,\mathrm{e}^{\mathrm{i}pz} + C_2\,\mathrm{e}^{-\mathrm{i}pz}, \qquad z < 0, \tag{11}$$

$$\Psi(z) = C_3\,\mathrm{e}^{\mathrm{i}pz}, \qquad z > 0, \quad p > 0, \quad p^2 = \frac{\omega^2}{c^2} - k^2. \tag{12}$$

The matching conditions (9) give the relations between the constants C_i ($i = 1, 2, 3$), which define the reflection and transmission coefficients

$$\mathcal{R}^{TE} \equiv \frac{C_2}{C_1} = \frac{-\mathrm{i}q}{p + \mathrm{i}q} = \mathrm{i}\sin\eta\,\mathrm{e}^{\mathrm{i}\eta}, \qquad \mathcal{T}^{TE} = \frac{p}{p + \mathrm{i}q} = \cos\eta\,\mathrm{e}^{\mathrm{i}\eta},$$

Here $\eta(p) = -\arctan(p/q)$, $p > 0$, stands for the TE-phase shift which determines the scattering matrix $S(p) = \exp[2\mathrm{i}\eta(p)]$.

Between the TE-modes there is no solution which decreases when $|z| \to \infty$. Thus, the TE-modes have the spectrum

$$\frac{\omega^2(\mathbf{k}, p)}{c^2} = k^2 + p^2, \qquad \mathbf{k} \in R^2, \qquad 0 \leqslant p < \infty. \tag{13}$$

Here the contribution k^2 is due to the free waves propagating in directions parallel to the plasma layer, and p^2 corresponds to the one-dimensional scattering in the normal direction with the phase shift $\eta(p)$.

Proceeding in the same way one obtains the reflection and transmission coefficients for the scattering states of the TM-modes

$$\mathcal{R}^{TM} = \frac{\mathrm{i}pq}{p^2 + k^2 + \mathrm{i}pq} = -\mathrm{i}\sin\mu\,\mathrm{e}^{\mathrm{i}\mu}, \qquad \mathcal{T}^{TM} = \frac{p^2 + k^2}{p^2 + k^2 + \mathrm{i}pq} = \cos\mu\,\mathrm{e}^{\mathrm{i}\mu}.$$

The corresponding scattering matrix $S(p, k) = \exp[2\mathrm{i}\mu(p, k)]$ is defined by the phase shift $\mu(p, k) = -\arctan(pq/(p^2 + k^2))$.

The solution decreasing when $|z|$ increases is present in the TM-spectrum as well. In this case the function $\Phi(z)$ is defined by the equation

$$\Phi''(z) - \left(k^2 - \frac{\omega^2}{c^2}\right)\Phi(z) = 0, \qquad k^2 - \frac{\omega^2}{c^2} \equiv \kappa^2 > 0. \tag{14}$$

The solution we are interested in should have the form

$$\Phi(z) = C_1\,\mathrm{e}^{\kappa z}, \qquad z < 0, \qquad \Phi(z) = C_2\,\mathrm{e}^{-\kappa z}, \qquad z > 0, \tag{15}$$

where $\kappa = +\sqrt{k^2 - \omega^2/c^2} > 0$. The matching conditions (8) lead to the equation for κ: $\kappa^2 + \kappa q - k^2 = 0$, the positive root of which is

$$\kappa = \sqrt{\frac{q^2}{4} + k^2} - \frac{q}{2}. \tag{16}$$

For the respective frequency squared we derive

$$\frac{\omega_{\mathrm{sp}}^2}{c^2} = k^2 - \kappa^2 = \frac{q}{2}\left(\sqrt{q^2 + 4k^2} - q\right) \geqslant 0, \qquad \mathbf{k} \in R^2. \tag{17}$$

Therefore, the frequency of the surface plasmon is real, and the solution obtained oscillates in time instead of being damped.

3. Spectral functions in a flat plasma sheet model

In this section we are going to construct the spectral functions of the model, namely, the spectral zeta function [6] and the (integrated) heat kernel [7]

$$\zeta(s) = \operatorname{Tr} L^{-s} = \sum_n \lambda_n^{-s}, \qquad K(t) = \operatorname{Tr}(\mathrm{e}^{-tL}) = \sum_n \mathrm{e}^{-\lambda_n t}, \tag{18}$$

where L is the differential operator and λ_n are the eigenvalues. The auxiliary variable t, $0 \leqslant t < \infty$, has the dimension (length)^2. In (18) the summation over the free waves reduces to the integration over the tangential wave vector $(2\pi)^{-1} \int_0^\infty k\, \mathrm{d}k \ldots$. The integration over the scattering states is carried out with the spectral density $\rho(p)$ (the function of the phase shift $\delta(p)$):

$$\rho(p) = \frac{1}{2\pi \mathrm{i}} \frac{\mathrm{d}}{\mathrm{d}p} \ln S(p) = \frac{1}{\pi} \frac{\mathrm{d}}{\mathrm{d}p} \delta(p). \tag{19}$$

For TE- and TM-modes the phase shift is given by $\eta(p)$ and $\mu(p)$, respectively.

3.1. TE-modes

For the spectral zeta function in the TE-sector of the model we get

$$\zeta^{\mathrm{TE}}(s) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \int_0^\infty \mathrm{d}p (k^2 + p^2)^{-s} \frac{1}{\pi} \frac{\mathrm{d}}{\mathrm{d}p} \eta(p), \qquad \eta(p) = -\arctan \frac{p}{q}. \tag{20}$$

The integral representation (20) is defined in the semi-plane $\operatorname{Re} s > 1$ and can be analytically extended all over the complex plane s save for separate points [1]. This is achieved by expressing the integrals in equation (20) in terms of the gamma-functions

$$\zeta^{\mathrm{TE}}(s) = \frac{q^{2-2s}}{8\pi^2} \frac{\Gamma(3/2 - s)\Gamma(s - 1/2)\Gamma(s - 1)}{\Gamma(s)} = \frac{q^{2-2s}}{8\pi(1 - s)\cos(\pi s)}. \tag{21}$$

The integrated heat kernel for TE-modes is given by

$$K^{\mathrm{TE}}(t) = K_0^{(d=2)}(t) \cdot K^{(d=1)}(t), \tag{22}$$

where $K_0^{(d=2)}(t)$ is the free two-dimensional heat kernel for the free waves propagating in directions parallel to the plasma layer

$$K_0^{(d=2)}(t) = \int_0^\infty \frac{k\, \mathrm{d}k}{2\pi} \mathrm{e}^{-k^2 t} = \frac{1}{4\pi t}, \tag{23}$$

and

$$K^{(d=1)}(t) = \frac{1}{\pi} \int_0^\infty \mathrm{d}p\, \mathrm{e}^{-p^2 t} \frac{\mathrm{d}}{\mathrm{d}p} \arctan\left(-\frac{q}{p}\right) = \frac{1}{2} \mathrm{e}^{tq^2} \operatorname{erfc}(q\sqrt{t}), \tag{24}$$

where $\operatorname{erfc}(q\sqrt{t})$ is the probability integral [8].

The structure of divergences in quantum field theory is determined by the coefficients of the asymptotic expansion of the heat kernel when $t \to +0$

$$K(t) = (4\pi t)^{-d/2} \sum_{n=0,1,2,\ldots} t^{n/2} B_{n/2} + \mathrm{ES}, \tag{25}$$

where d is the dimension of space, ES stands for the exponentially small corrections.

For the TE-modes we get from (22)–(24)

$$\begin{aligned} &B_0 = 0, \quad B_{1/2} = \sqrt{\pi}, \quad B_1 = -2q, \quad B_{3/2} = \sqrt{\pi} q^2, \quad B_2 = -\frac{4}{3} q^3, \\ &B_{5/2} = \frac{\sqrt{\pi}}{2} q^4, \quad B_3 = -\frac{8}{15} q^5, \quad B_{7/2} = \frac{\sqrt{\pi}}{6} q^6, \quad \ldots. \end{aligned} \tag{26}$$

3.2. TM-modes

In the TM-modes both the photon and the surface plasmon branches of the spectrum are present.

With equations (19) and the TM = phase shift in we obtain the photon zeta function

$$\zeta_{\mathrm{ph}}^{\mathrm{TM}}(s) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \int_0^\infty \mathrm{d}p (k^2+p^2)^{-s} \frac{1}{\pi} \frac{\mathrm{d}}{\mathrm{d}p} \mu(p,k)$$
$$= \frac{q}{2\pi^2} \int_0^\infty k\,\mathrm{d}k \int_0^\infty \frac{\mathrm{d}p}{(k^2+p^2)^s} \frac{p^2-k^2}{(k^2+p^2)^2+q^2p^2}. \tag{27}$$

After several changes of the variables (see [1]) the double integral (27) is analytically extended to the whole complex s plane except separate points. This yields

$$\zeta_{\mathrm{ph}}^{\mathrm{TM}}(s) = \frac{q^{2-2s}}{8\pi^2} \frac{s\Gamma(3/2-s)\Gamma(s-1/2)}{(s-1)(2-s)} = \frac{1}{8\pi} \frac{q^{2-2s}s}{(1-s)(2-s)\cos\pi s}. \tag{28}$$

The heat kernel for the photon branch of the TM-modes reads

$$K_{\mathrm{ph}}^{\mathrm{TM}}(t) = \frac{q}{2\pi^2} \int_0^\infty k\,\mathrm{d}k\,\mathrm{e}^{-k^2t} \int_0^\infty \mathrm{d}p\,\mathrm{e}^{-p^2t} \frac{p^2-k^2}{(k^2+p^2)^2+q^2p^2}. \tag{29}$$

For deriving the coefficients of the heat kernel expansion at small t the following representation of the heat kernel is more convenient [8]:

$$K_{\mathrm{ph}}^{\mathrm{TM}}(t) = \frac{q}{2\pi^2} \int_0^1 \mathrm{d}x (2x^2-1) I(x), \tag{30}$$

where

$$I(x) = \int_0^\infty \frac{r^2\,\mathrm{e}^{-r^2t}}{r^2+q^2x^2}\,\mathrm{d}r = \frac{1}{2}\sqrt{\frac{\pi}{t}} - \frac{\pi q x}{2}\left[\mathrm{e}^{q^2x^2t} - \frac{2}{\sqrt{\pi}} \sum_{k=0}^\infty \frac{2^k (qx\sqrt{t})^{2k+1}}{(2k+1)!!}\right].$$

Then the coefficients for the photon sector of the TM-modes are

$$B_0 = 0, \quad B_{1/2} = 0, \quad B_1 = -\frac{2}{3}q, \quad B_{3/2} = 0, \quad B_2 = \frac{4}{15}q^3,$$
$$B_{5/2} = -\frac{\sqrt{\pi}}{6}q^4, \quad B_3 = \frac{8}{35}q^5, \quad B_{7/2} = -\frac{\sqrt{\pi}}{12}q^6. \tag{31}$$

From the definition of the spectral zeta function (18) for the surface plasmon branch of the spectrum (17) it follows that

$$\zeta_{\mathrm{sp}}^{\mathrm{TM}}(s) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{\omega_{\mathrm{sp}}^{-2s}(\mathbf{k})}{c^{-2s}} = \left(\frac{2}{q}\right)^s \int_0^\infty \frac{k\,\mathrm{d}k}{2\pi} \left(\sqrt{q^2+4k^2}-q\right)^{-s}. \tag{32}$$

The convergence of this integral in the region $k \to 0$ requires $\mathrm{Re}\,s < 1$, but when $k \to \infty$ it exists only if $\mathrm{Re}\,s > 2$. Thus for the spectrum (17) it is impossible to construct the zeta function by making use of the analytical continuation method, since one cannot define this function in any finite domain of the complex plane s.

However it turns out that for this branch of the spectrum the heat kernel can be constructed explicitly. Indeed, by making use of equations (17), (18) we obtain

$$K_{\mathrm{sp}}^{\mathrm{TM}}(t) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \exp\left[-\frac{\omega_{\mathrm{sp}}^2(\mathbf{k})}{c^2}t\right] = \int_0^\infty \frac{k\,\mathrm{d}k}{2\pi} \exp\left[-\frac{q}{2}\left(\sqrt{q^2+4k^2}-q\right)t\right]$$
$$= \frac{1}{2\pi q^2 t^2} + \frac{1}{4\pi t}. \tag{33}$$

The first term in the second line of equation (33) is absent in the standard expansion (25), and the second term yields $B_{1/2} = 2\sqrt{\pi}$. The rest of coefficients $B_{n/2}$ with $n \neq 1$ equal zero. Thus the surface plasmon with the spectrum (17) is a simple physical model that has no spectral zeta function, but possesses the integrated heat kernel with a nonstandard asymptotic expansion.

4. Local heat kernel

The local heat kernel for the TE-sector of the model is defined by

$$K(\mathbf{r}, \mathbf{r}'; t) = \sum_n \varphi_n^*(\mathbf{r})\varphi_n(\mathbf{r}')\, \mathrm{e}^{-\lambda_n t}, \qquad L\varphi_n(\mathbf{r}') = \lambda_n\varphi_n(\mathbf{r}'), \quad L = -\Delta. \tag{34}$$

It obeys the heat conduction equations with respect to the variables $(\mathbf{r}, t)$ and $(\mathbf{r}', t)$

$$\left(\Delta_{\mathbf{r}} - \frac{\partial}{\partial t}\right) K(\mathbf{r}, \mathbf{r}'; t) = 0, \qquad \left(\Delta_{\mathbf{r}'} - \frac{\partial}{\partial t}\right) K(\mathbf{r}, \mathbf{r}'; t) = 0 \tag{35}$$

and the initial condition $K(\mathbf{r}, \mathbf{r}'; t) \to \delta(\mathbf{r} - \mathbf{r}')$, when $t \to 0^+$.

For the heat kernel of the TE-modes it is easy to show that

$$K(\mathbf{r}, \mathbf{r}'; t) = K_0^{(d=2)}(\mathbf{s}, \mathbf{s}'; t) \cdot K(z, z'; t), \qquad \mathbf{r} = (\mathbf{s}, z), \quad \mathbf{s} = (x, y), \tag{36}$$

where $K_0^{(d=2)}(\mathbf{s}, \mathbf{s}'; t) = (4\pi t)^{-1} \exp[-(\mathbf{s} - \mathbf{s}')^2/4t]$ is the free heat kernel in the directions parallel to the plane $z = 0$.

To derive the heat kernel $K(z, z'; t)$ the integral equations are used, which naturally arise when we seek the Green function as a heat potential of a simple or double layer [9].

Then it is convenient to consider four components of the local heat kernel $K(z, z'; t)$ depending on the values of its arguments: $K_{-+}(z, z'; t), z < 0, z' > 0$; $K_{++}(z, z'; t), z, z' > 0$; $K_{+-}(z, z'; t), z > 0, z' < 0$; $K_{--}(z, z'; t), z, z' < 0$.

The components of the heat kernel we represent as the heat potentials of the simple layers [10]

$$K_{-+}(z, z'; t) = \int_0^t \mathrm{d}\tau\, K_0(z, 0; t - \tau)\alpha_1(\tau; z'), \qquad z < 0, \tag{37}$$

$$K_{++}(z, z'; t) = K_0(z, z'; t) + \int_0^t \mathrm{d}\tau\, K_0(z, 0; t - \tau)\alpha_2(\tau; z'), \qquad z > 0, \tag{38}$$

where $\alpha_1(\tau; z')$ and $\alpha_2(\tau; z')$ are the densities of the heat potentials to be found, and $K_0(z, z'; t) = (4\pi t)^{-1/2} \exp[-(z - z')^2/4t]$ is the free heat potential on an infinite line.

Substituting equations (37) and (38) into the matching conditions (9) we obtain

$$\int_0^t \mathrm{d}\tau\, K_0(0, 0; t - \tau)(\alpha_1 - \alpha_2) = K_0(0, z'; t), \quad z' > 0 \tag{39}$$

$$2\frac{\partial}{\partial z} K_0(z, z'; t)|_{z=0} - 4q\int_0^t \mathrm{d}\tau\, K_0(0, 0; t - \tau)\alpha_1 = (\alpha_1 + \alpha_2), \quad z' > 0. \tag{40}$$

Here we have omitted the arguments of $\alpha_i = \alpha_i(\tau; z'), i = 1, 2$, and have taken into account that at the interface the derivative of each single layer potential has a jump equal to α_i [9].

Applying the Laplace transform to the integral equations (39), (40) we get the Laplace images of the densities α_1 and α_2 and hence the images of K_{-+} and K_{++}. The inverse Laplace transform gives the corresponding Green functions. By making use of the same technique one can derive the components K_{--} and K_{--}.

All the four components of the heat kernel $K(z, z'; t)$ assume the form

$$K(z, z'; t) = K_0(z, z'; t) - \frac{q}{2}\, \mathrm{e}^{q(|z|+|z'|)+q^2 t} \mathrm{erfc}\left(q\sqrt{t} + \frac{|z|+|z'|}{2\sqrt{t}}\right),$$
$$-\infty < z, \qquad z' < \infty, \qquad z \neq 0, \qquad z' \neq 0. \tag{41}$$

The representation (41) appeared in the paper [11] where the quantum field theoretical aspects of the delta potential were considered systematically.

For the problem under consideration the heat potentials afford a more transparent method for deriving the heat kernel.

In the TM-sector of the model the construction of the local heat kernel proves to be more complicated. The factorization equation (36) no longer holds. To apply the technique of integral equations [10] one has to get rid of the spectral parameter $(\omega(p, \mathbf{k})/c)^2$ in the right-hand side of the first matching condition (8). From the definition of the heat kernel and the heat conduction equation, it follows that this matching condition can be obviously rewritten in the form $[\Delta_{\mathbf{r}} K(\mathbf{s}, z = 0, \mathbf{r}'; t)] = 2q\partial_z K(\mathbf{s}, z = 0, \mathbf{r}'; t)$.

An interesting spectral problem arises here when we confine ourself to the one-dimensional problem with $(\omega(p)/c)^2 = p^2$:

$$-\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) = p^2\varphi(z), \qquad -\infty < z < \infty, \qquad z \neq 0, \tag{42}$$

$$[\varphi'(z = 0)] = 0, [\varphi''(z = 0)] = 2q\varphi'(z = 0). \tag{43}$$

At first sight, this spectral problem is completely different from the analogous one for the δ potential (see equations (7), (10) and (9)). However the respective eigenfunctions $\varphi_p(z)$ and $\Phi_p(z)$ are connected by the relation $\varphi'_p(z) = \Phi_p(z)$. Both problems have the same positive continuous spectrum $0 < p^2 < \infty$. Furthermore the respective phase shifts, and consequently the scattering matrices, coincide. From here we infer immediately that these spectral problems have the same integrated heat kernels defined by equation (24), while the local heat kernels are obviously different.

5. Conclusion

The plasma sheet model investigated here proves to be very instructive from the standpoint of spectral analysis. The spectrum of the model contains both continuous branches and bound states (surface plasmon). It is remarkable that for the latter the spectral zeta function cannot be constructed, at least by the standard analytic continuation method. At the same time the integrated heat kernel is found in an explicit form for all the branches of the spectrum. On the whole this heat kernel of the model has the asymptotic expansion of a non-canonical form.

By making use of the heat potentials the local heat kernel in the TE-sector of the model is derived. For the heat equation on an infinite line with the δ-source a nontrivial counterpart is found, namely, a spectral problem with point interaction that possesses the same integrated heat kernel. However the local heat kernels in these spectral problems are different.

The spectral analysis of the same model describing the plasma layers of different geometry, for example, circular infinite cylinder or sphere, is of interest also.

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